Instructor: TBD
Quantitative Theory and Methods
email: XXX@emory.edu
Office:
Office Hours:
TA: TBD

QTM 501: Math and Stats Foundations

Class: XXX

Credit hours: 4

Student contact hours: 5

After introducing the calculus and linear algebra necessary to provide a mathematical introduction to probability and statistics, this course will cover the fundamentals of probability theory and introduce sampling, the distribution of basic sample statistics, and the problem of estimation and inference.

COURSE OBJECTIVES

By the end of this course, students should be able to

- Produce or interpret a variety of visualizations of multivariable functions.
- Compute rates of change of multivariable functions.
- Find maximum and minimum values of a multivariable function, including with constraint. • Discuss the most common probability distributions, both discrete and continuous.
- Compute probability using integrals
- Compute expectation using integration by parts and series
- Formally describe the distributions of some sample statistics.
- Use statistical tools to make inferences on the mean and proportion
- Discuss the problem of estimation using statistical language and tools
- Understand and be able to state formally the key probabilistic and statistical results underlying subsequent courses in the program.
-

CLASS REQUIREMENTS

Grades will be based on

- Four homework assignments (40 %)
- Four quizzes (20 %)
- Final exam (30 %)
- Class participation and attendance (10%)

HOMEWORK

There will be weekly homework assignments. The homework assignments will consist of 2 analytical problem sets and 2 writing based problem sets. Each assignment will have its due date indicated and should be submitted through Canvas. Usually it would be due before the class (11:30 am). Any assignment submitted after the due date/time will be considered 0 points. To accommodate unexpected circumstances,

your lowest homework grade will be automatically dropped at the end of the semester. Working together on the homework assignments is encouraged, but you must write your own answers. It is highly recommended that you make your solo effort on all the problems before consulting others.

Below is an example homework.

Homework 1

Writing Problem Set 1: Multivariable functions

Recall from Calculus I the vertical line test: a continuous curve in the plane is the graph of a function if any vertical line (parallel to y-axis) meets the curve in exactly one point.

1. What is the analogous test for surfaces in three-dimensional space to be the graph of a function of two variables?
2. In particular, describe how a surface which passes your test defines a function.
3. Suppose you are looking at 2017 World bank data for countries around the world. Two of the variables in the dataset are GDP (in 2017) and per capita income. Consider the assignment which maps gdp and per capita income to population size. Is this a function? Why or why not.

Writing Problem Set 2: Derivatives

Recall that one can compute derivatives numerically and symbolically.

1. Explain in words something you found about the relationship between the size of delta_x and the approximation to the derivative f_x. Is the relationship linear, quadratic, or does it depend on f? 2. As we explained in lecture, when you are climbing a hill which happens to be the graph of a function f(x,y) the gradient vector of f points in the direction of steepest ascent. Explain why if you shuffle your foot forward a tiny step this is true, but if you take a big step, then this might fail.
3. Compute the derivative of f(x) = exp(x) when x = 1 using delta_x = 1/10^k in your approximation. For which value k less than or equal to 10 do you have the best precision (knowing exp(1) = 2.71828182845905)? Hint: the answer is not 10! Try to explain why.

Analytical problem set 1:

Problems from Calculus Early Transcendentals
Section 9.1: Vocabulary check, 1, 23, 33-36, 37, 41, 51, 59, 65, 73, 99, 103, 113, 119.
Section 9.2: Vocabulary check, 1, 3, 49-52, 57, 67, 97.
Section 9.3: Vocabulary check, 1, 43-46, 53-61, 79, 81, 121, 123.
Section 9.4: Vocabulary check, 41, 43.
Section 9.5: Vocabulary check, 1, 7, 11, 15, 35, 39, 47, 69, 75, 77.
Section 12.1: 1, 11, 23 (and sketch), 29 (and sketch), 35, 37.

Analytical problem set 2:

Problems from Calculus Early Transcendentals
Section 12.6: 3, 5, 21-28, 30, 32.
Section 14.1: 3, 4, 5, 29, 32, 36, 39.
Section 14.3: 5, 10, 11 (sketch as done in class), 15, 17, 19, 47, 63, 74, 75, 79, 84, 86.
Section 14.4: 1, 19, 25.
Section 14.5: 1, 7, 42, 49.
Section 14.6: 21, 34 (omit (b)), 38.

Section 14.7: 1, 5 (omit graphing), 51, 59.
Section 14.8: 3, 19, 27.

Section 15.1: 6, 7, 9.
Section 15.2: 3, 15, 25, 28.
Section 15.3: 1-6, 35, 40.
Section 15.4: 27, 29, 30.
Section 15 Review: True-False Quiz 1-7 and Exercises 51, 52.

EXAMS

The final will be a comprehensive in-class exam. **No collaboration is allowed on the exams**.

Important Dates

Problem Set 1: Week 1

Problem Set 2: Week 2

Problem Set 3: Week 3

Problem Set 4: Week 4

Quiz 1: Week 1

Quiz 2: Week 2
Quiz 3: Week 3
Quiz 4: Week 4

Final: TBD

HONOR CODE

All students enrolled at Emory are expected to abide by the Emory College Honor Code. Any type of academic misconduct is not allowed which includes 1) receiving or giving information about the content or conduct of an ex- amination knowing that the release of such information is not allowed and 2) plagiarizing, whether intentionally or unintentionally, in any assignment. For the activities that are considered to be academically dishonest, refer to the Honor Code: http://catalog.college.emory.edu/academic/policies-regulations/honor code.html.

DISABILITY ACCOMMODATIONS

If you are seeking classroom accommodations or academic adjustments under the Americans with

Disabilities Act, you are required to register with Office of Accessibility Services (http://accessibility.emory.edu). To receive academic accommodations for this class, please obtain the relevant letter and meet with me at the beginning of the semester. Students are expected to give two weeks notice of the need for accommodations.

REQUIRED TEXTBOOKS

• Calculus Early Transcendentals, James Stewart 8th edition, ISBN 978-1-285-74155-0 •
Linear algebra and its applications, 5th Edition, David C. Lay
• An introduction to mathematical statistics and its applications, sixth Edition, by Richard J. Larsen C. and Morris L. Marx, 2017.

LECTURE PLAN

Week 1

Topics:

Single variable calculus
- Sequences and series for probability
- Integration by parts

Multivariable calculus
- Graphing Multivariable functions
- Partial derivatives omitting epsilon-delta definition of limit
- Optimization and 2nd derivative test
- Double integrals and computing integrals over non-rectangular regions

Reading:

[Stewart] Sections 1.1,1.2, 7.1, 11.1, 11.2, 14.1, 14.3, 14.5, 14.6, 14.7, 15.1, 15.2, 15.4

Week 2

Topics:

Matrices
- Systems of linear equations
- Vector equations
- Determinants

Matrix operations
- Inverse of a matrix
- Characterizations of invertible matrices

Least-Squares Problems

- Normal equations
- Applications to linear models

Reading:

[Lay] Sections 1.1, 1.2, 1.3, 1.4, 1.9, 2.1, 2.2, 2.3, 3.1, 3.2, 3.3, 6.5, 6.6

Week 3

Topics:

Probability foundations
- Sample spaces
- Conditional probability
- Independence
- Combinatorial probability
- Binomial probabilities
- Discrete random variables

Calculus based probability
- Continuous random variables
- Expected values
- Variance
- Joint densities
- Properties of mean and variance

Distributions
- Normal distribution
- Central limit theorem
- Other distributions

Reading:

[Larsen and Marx] Sections 2.2, 2.3, 2.4, 2.5, 3.2, 3.3, 3.4, 3.5, 3.6, 3.7, 3.9, 4.2, 4.3

Week 4

Topics:

Estimation
- Maximum likelihood
- Method of moments
- Interval estimation
- Properties of estimators

Inference
- Hypothesis testing

- Testing binomial data
- Type I and type II errors
- Drawing inferences about the mean
- Two sample problems

Reading:

[Larsen and Marx] Sections 5.1, 5.2, 5.3, 5.4, 5.5, 5.7, 6.1, 6.2, 6.3, 6.4, 7.4, 9.2, 9.4, 9.5